

# Lecture: Basic Elements

Lutz Kruschwitz & Andreas Löffler

Stochastic Discounted Cash Flow, Section 2.1

**Remark:** The slightly expanded second edition ([Springer](#), open access) has different enumeration than the first ([Wiley](#)). We use Springer's enumeration in the slides and Wiley's in the videos.



# Outline

## 1 Introduction

DCF

The predecessors

## 2.1 Fundamental terms

2.1.1 Cash flows

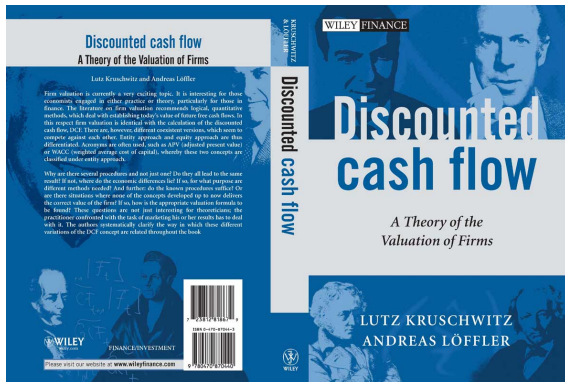
2.1.2 Taxes

2.1.3 Cost of capital

2.1.4 Time

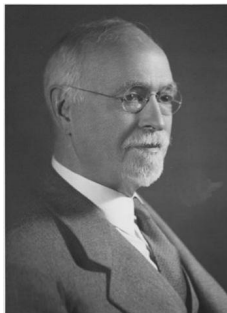
## Summary





**Remark:** I have recorded the videos using the first edition (Wiley). Recording again only because of different enumerations in the second edition (Springer) was too much for me . . .





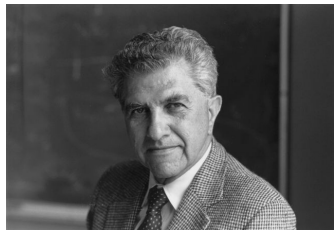
Fisher is one of the earliest American Neoclassicals. He studied Mathematics, Social Science and Philosophy. 1892 Professor at Yale.





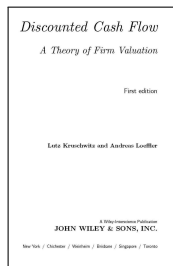
Modigliani was born in Italy, moved to USA in 1939. 1962 Professor at Massachusetts Institute of Technology. 1985 Nobel Laureate in Economics.





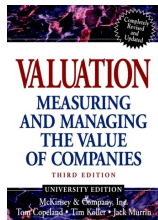
1961 Professor at University of Chicago. 1990 Nobel Laureate in Economics.





1. To put **taxes and uncertainty together** into one model and
2. To give **precise formal definitions** of several concepts such as
  - ▶ cash flows (gross, net, free, ...?)
  - ▶ taxes (firm income, personal income or both, ...?)
  - ▶ cost of capital (discount rates, returns, ...?)
3. While **maintaining** the following **principles**:
  - ▶ no free lunch (goes back to Modigliani–Miller!)
  - ▶ no revelation of stochastic structure of future cash flows.





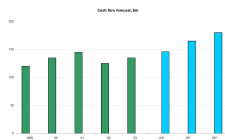
Copeland/Koller/Murrin

Valuation based on discounted cash flow (DCF) involves discounting

- ▶ of future payment surpluses
- ▶ after consideration of taxes
- ▶ using appropriate cost of capital.







CF forecast

What matters are **future** cash flows.

But, the question of **how to forecast cash flows** will not be considered here,

nor the question of how to derive **cash flows from balance sheets**.

Furthermore, the investment policy (expansion and replacement investments) will be given.





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$$\begin{array}{r} \text{EBIT} \\ + \text{ Accruals} \\ \hline = \text{ Gross cash flows before taxes} \\ - \text{ Corporate income taxes} \\ - \text{ Investment expenses} \\ \hline = \text{ Free cash flow} \\ - \text{ Interest, debt service} \\ - \text{ dividends, capital reduction} \\ \hline = \text{ Zero} \end{array}$$





US Tax

Authority

We consider two different types of income tax:

- Corporate income tax (Chapter 2).
- Personal income tax (Chapter 3).

Value-based and sales taxes are ignored.



Characteristics are

- the tax subject (who?)
- the tax object (why?)
- the tax due (how much?), which is a product of the tax base and a linear tax scale.

The image shows a screenshot of a German tax form titled 'Umsatzsteuererklärung' (Sales Tax Declaration) for the year 2003. The form is divided into several sections. At the top, there is a header with the year '2003' and a small table with columns for 'Steuernummer', 'Steuernummer', 'Steuernummer', and 'Steuernummer'. Below this, there are fields for 'Umsatzsteuererklärung' and 'Umsatzsteuererklärung'. The form also includes a section for 'Umsatzsteuererklärung' and a section for 'Umsatzsteuererklärung'. The form is a standard German tax form used for reporting sales tax.

German tax file

Notice that in our model the **tax rate is deterministic.**



Instrument	Bid	Ask	Volume
134.100 Euro area 3m swap	0.00	0.00	0.00
134.100 Euro area 6m swap	0.00	0.00	0.00
134.100 Euro area 9m swap	0.00	0.00	0.00
134.100 Euro area 12m swap	0.00	0.00	0.00
134.100 Euro area 15m swap	0.00	0.00	0.00
134.100 Euro area 18m swap	0.00	0.00	0.00
134.100 Euro area 21m swap	0.00	0.00	0.00
134.100 Euro area 24m swap	0.00	0.00	0.00
134.100 Euro area 27m swap	0.00	0.00	0.00
134.100 Euro area 30m swap	0.00	0.00	0.00
134.100 Euro area 33m swap	0.00	0.00	0.00
134.100 Euro area 36m swap	0.00	0.00	0.00
134.100 Euro area 39m swap	0.00	0.00	0.00
134.100 Euro area 42m swap	0.00	0.00	0.00
134.100 Euro area 45m swap	0.00	0.00	0.00
134.100 Euro area 48m swap	0.00	0.00	0.00
134.100 Euro area 51m swap	0.00	0.00	0.00
134.100 Euro area 54m swap	0.00	0.00	0.00
134.100 Euro area 57m swap	0.00	0.00	0.00
134.100 Euro area 60m swap	0.00	0.00	0.00
134.100 Euro area 63m swap	0.00	0.00	0.00
134.100 Euro area 66m swap	0.00	0.00	0.00
134.100 Euro area 69m swap	0.00	0.00	0.00
134.100 Euro area 72m swap	0.00	0.00	0.00
134.100 Euro area 75m swap	0.00	0.00	0.00
134.100 Euro area 78m swap	0.00	0.00	0.00
134.100 Euro area 81m swap	0.00	0.00	0.00
134.100 Euro area 84m swap	0.00	0.00	0.00
134.100 Euro area 87m swap	0.00	0.00	0.00
134.100 Euro area 90m swap	0.00	0.00	0.00
134.100 Euro area 93m swap	0.00	0.00	0.00
134.100 Euro area 96m swap	0.00	0.00	0.00
134.100 Euro area 99m swap	0.00	0.00	0.00
134.100 Euro area 102m swap	0.00	0.00	0.00
134.100 Euro area 105m swap	0.00	0.00	0.00
134.100 Euro area 108m swap	0.00	0.00	0.00
134.100 Euro area 111m swap	0.00	0.00	0.00
134.100 Euro area 114m swap	0.00	0.00	0.00
134.100 Euro area 117m swap	0.00	0.00	0.00
134.100 Euro area 120m swap	0.00	0.00	0.00
134.100 Euro area 123m swap	0.00	0.00	0.00
134.100 Euro area 126m swap	0.00	0.00	0.00
134.100 Euro area 129m swap	0.00	0.00	0.00
134.100 Euro area 132m swap	0.00	0.00	0.00
134.100 Euro area 135m swap	0.00	0.00	0.00
134.100 Euro area 138m swap	0.00	0.00	0.00
134.100 Euro area 141m swap	0.00	0.00	0.00
134.100 Euro area 144m swap	0.00	0.00	0.00
134.100 Euro area 147m swap	0.00	0.00	0.00
134.100 Euro area 150m swap	0.00	0.00	0.00
134.100 Euro area 153m swap	0.00	0.00	0.00
134.100 Euro area 156m swap	0.00	0.00	0.00
134.100 Euro area 159m swap	0.00	0.00	0.00
134.100 Euro area 162m swap	0.00	0.00	0.00
134.100 Euro area 165m swap	0.00	0.00	0.00
134.100 Euro area 168m swap	0.00	0.00	0.00
134.100 Euro area 171m swap	0.00	0.00	0.00
134.100 Euro area 174m swap	0.00	0.00	0.00
134.100 Euro area 177m swap	0.00	0.00	0.00
134.100 Euro area 180m swap	0.00	0.00	0.00
134.100 Euro area 183m swap	0.00	0.00	0.00
134.100 Euro area 186m swap	0.00	0.00	0.00
134.100 Euro area 189m swap	0.00	0.00	0.00
134.100 Euro area 192m swap	0.00	0.00	0.00
134.100 Euro area 195m swap	0.00	0.00	0.00
134.100 Euro area 198m swap	0.00	0.00	0.00
134.100 Euro area 201m swap	0.00	0.00	0.00
134.100 Euro area 204m swap	0.00	0.00	0.00
134.100 Euro area 207m swap	0.00	0.00	0.00
134.100 Euro area 210m swap	0.00	0.00	0.00
134.100 Euro area 213m swap	0.00	0.00	0.00
134.100 Euro area 216m swap	0.00	0.00	0.00
134.100 Euro area 219m swap	0.00	0.00	0.00
134.100 Euro area 222m swap	0.00	0.00	0.00
134.100 Euro area 225m swap	0.00	0.00	0.00
134.100 Euro area 228m swap	0.00	0.00	0.00
134.100 Euro area 231m swap	0.00	0.00	0.00
134.100 Euro area 234m swap	0.00	0.00	0.00
134.100 Euro area 237m swap	0.00	0.00	0.00
134.100 Euro area 240m swap	0.00	0.00	0.00
134.100 Euro area 243m swap	0.00	0.00	0.00
134.100 Euro area 246m swap	0.00	0.00	0.00
134.100 Euro area 249m swap	0.00	0.00	0.00
134.100 Euro area 252m swap	0.00	0.00	0.00
134.100 Euro area 255m swap	0.00	0.00	0.00
134.100 Euro area 258m swap	0.00	0.00	0.00
134.100 Euro area 261m swap	0.00	0.00	0.00
134.100 Euro area 264m swap	0.00	0.00	0.00
134.100 Euro area 267m swap	0.00	0.00	0.00
134.100 Euro area 270m swap	0.00	0.00	0.00
134.100 Euro area 273m swap	0.00	0.00	0.00
134.100 Euro area 276m swap	0.00	0.00	0.00
134.100 Euro area 279m swap	0.00	0.00	0.00
134.100 Euro area 282m swap	0.00	0.00	0.00
134.100 Euro area 285m swap	0.00	0.00	0.00
134.100 Euro area 288m swap	0.00	0.00	0.00
134.100 Euro area 291m swap	0.00	0.00	0.00
134.100 Euro area 294m swap	0.00	0.00	0.00
134.100 Euro area 297m swap	0.00	0.00	0.00
134.100 Euro area 300m swap	0.00	0.00	0.00

Reuters monitor

It is obvious what the cost of capital is in a one-period context. In a multi-period context there are at least **three different notions** of this concept: cost of capital can be

- ▶ returns,
- ▶ discount rates, or
- ▶ yields.

How now?



First, let us ignore uncertainty.

Notation:

$FCF$  firm's free cash flow  
 $V$  value of the firm

Idea:

Cost of capital is used for **discounting** (we are very loose here), hence

$$V_0 = \frac{FCF_1}{1 + k_0} + \frac{FCF_2}{(1 + k_0)(1 + k_1)} + \dots$$



This idea shall also be applied in the future: at  $t = 1$  we want to have

$$V_1 = \frac{FCF_2}{1 + k_1} + \frac{FCF_3}{(1 + k_1)(1 + k_2)} + \dots$$

where  $k_1$  is the **same cost of capital from the last slide!**



Then the definition of cost of capital should run

$$k_t \stackrel{\text{Def}}{=} \frac{FCF_{t+1} + V_{t+1}}{V_t} - 1$$

Implication: Costs of capital are inevitably (expected) returns.





A different approach could be

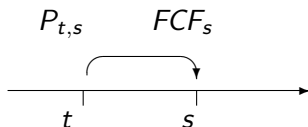
$$V_0 = \frac{FCF_1}{1 + k_0} + \frac{FCF_2}{(1 + k_1)^2} + \dots$$

but then  $\implies V_1 \stackrel{?}{=} \frac{FCF_2}{1 + k_1} + \frac{FCF_3}{(1 + k_2)^2} + \dots$

Here the costs of capital are **yields**. We do not think much along this course (this is a different concept), because it is difficult to observe yields empirically.



You pay at time  $t$  a price  $P_{t,s}$  for cash flow  $FCF_s$  due at  $s$ :



We would then define a discount rate as

$$P_{t,s} \stackrel{\text{Def}}{=} \frac{FCF_s}{(1 + \kappa_{t,s})^{s-t}}$$

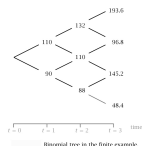
What relation exists between these discount rates and (expected) returns (=cost of capital)?

Will be understood later...

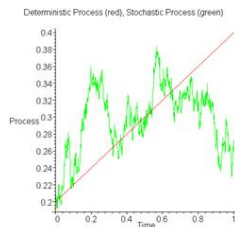


## Different notions of time

discrete (easy to handle)



continuous (elegant, but laborious)



## Time horizon

- ▶ finite
- ▶ infinite: Here we assume transversality, which is equivalent to saying 'nothing strange happens when  $T \rightarrow \infty$ '.



Valuation requires knowledge of

- free cash flows,
- taxes,
- cost of capital.

Costs of capital are returns, not yields.

